

NAG Toolbox for MATLAB

g13fa

1 Purpose

g13fa estimates the parameters of either a standard univariate regression GARCH process, or a univariate regression-type I AGARCH(p, q) process (see Engle and Ng 1993).

2 Syntax

```
[theta, se, sc, covr, hp, et, ht, lgf, ifail] = g13fa(dist, yt, x, ip,
iq, nreg, mn, isym, theta, hp, copts, maxit, tol, 'num', num, 'npar',
npar)
```

3 Description

A univariate regression-type I AGARCH(p, q) process, with q coefficients α_i , for $i = 1, \dots, q$, p coefficients β_i , for $i = 1, \dots, p$, and k linear regression coefficients b_i , for $i = 1, \dots, k$, can be represented by:

$$y_t = b_o + x_t^T b + \epsilon_t \quad (1)$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i (\epsilon_{t-i} + \gamma)^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad t = 1, \dots, T \quad (2)$$

where $\epsilon_t | \psi_{t-1} = N(0, h_t)$ or $\epsilon_t | \psi_{t-1} = S_t(df, h_t)$. Here S_t is a standardized Student's t -distribution with df degrees of freedom and variance h_t , T is the number of terms in the sequence, y_t denotes the endogenous variables, x_t the exogenous variables, b_o the regression mean, b the regression coefficients, ϵ_t the residuals, h_t is the conditional variance, df the number of degrees of freedom of the Student's t -distribution, and ψ_t the set of all information up to time t .

g13fa provides an estimate for $\hat{\theta}$, the parameter vector $\theta = (b_o, b^T, \omega^T)$ where $b^T = (b_1, \dots, b_k)$, $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma)$ when **dist** = 'N' and $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma, df)$ when **dist** = 'T'.

isym, **mn** and **nreg** can be used to simplify the GARCH(p, q) expression in (1) as follows:

No Regression and No Mean

$$y_t = \epsilon_t,$$

$$\mathbf{isym} = 0,$$

$$\mathbf{mn} = 0,$$

$$\mathbf{nreg} = 0 \text{ and}$$

$$\theta \text{ is a } (p + q + 1) \text{ vector when } \mathbf{dist} = \text{'N'} \text{ and a } (p + q + 2) \text{ vector when } \mathbf{dist} = \text{'T'}.$$

No Regression

$$y_t = b_o + \epsilon_t,$$

$$\mathbf{isym} = 0,$$

$$\mathbf{mn} = 1,$$

$$\mathbf{nreg} = 0 \text{ and}$$

$$\theta \text{ is a } (p + q + 2) \text{ vector when } \mathbf{dist} = \text{'N'} \text{ and a } (p + q + 3) \text{ vector when } \mathbf{dist} = \text{'T'}.$$

Note: if the $y_t = \mu + \epsilon_t$, where μ is known (not to be estimated by g13fa) then (1) can be written as $y_t^\mu = \epsilon_t$, where $y_t^\mu = y_t - \mu$. This corresponds to the case **No Regression and No Mean**, with y_t replaced by $y_t - \mu$.

No Mean

$$y_t = x_t^T b + \epsilon_t,$$

$$\text{isym} = 0,$$

$$\text{mn} = 0,$$

$$\text{nreg} = k \text{ and}$$

θ is a $(p + q + k + 1)$ vector when **dist** = 'N' and a $(p + q + k + 2)$ vector, when **dist** = 'T'.

4 References

- Bollerslev T 1986 Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327
- Engle R 1982 Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008
- Engle R and Ng V 1993 Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777
- Hamilton J 1994 *Time Series Analysis* Princeton University Press

5 Parameters**5.1 Compulsory Input Parameters****1: dist – string**

The type of distribution to use for e_t .

dist = 'N'

A Normal distribution is used.

dist = 'T'

A Student's t -distribution is used.

Constraint: **dist** = 'N' or 'T'.

2: yt(num) – double array

The sequence of observations, y_t , for $t = 1, \dots, T$.

3: x(ldx,*) – double array

The first dimension of the array **x** must be at least **num**

The second dimension of the array must be at least $\max(1, \text{nreg} + \text{mn})$

Row t of **x** must contain the time dependent exogenous vector x_t , where $x_t^T = (x_t^1, \dots, x_t^k)$, for $t = 1, \dots, T$.

4: ip – int32 scalar

The number of coefficients, β_i , for $i = 1, \dots, p$.

Constraint: **ip** ≥ 0 (see also **npar**)

5: iq – int32 scalar

The number of coefficients, α_i , for $i = 1, \dots, q$.

Constraint: **iq** ≥ 1 (see also **npar**)

6: **nreg – int32 scalar**

k , the number of regression coefficients.

Constraint: **nreg** ≥ 0 (see also **npar**)

7: **mn – int32 scalar**

If **mn** = 1, the mean term b_0 will be included in the model.

Constraint: **mn** = 0 or 1.

8: **isym – int32 scalar**

If **isym** = 1 then the asymmetry term γ will be included in the model.

Constraint: **isym** = 0 or 1.

9: **theta(npar) – double array**

The initial parameter estimates for the vector θ .

The first element must contain the coefficient α_o and the next **iq** elements must contain the coefficients α_i , for $i = 1, \dots, q$.

The next **ip** elements must contain the coefficients β_j , for $j = 1, \dots, p$.

If **isym** = 1, the next element must contain the asymmetry parameter γ .

If **dist** = 'T', the next element must contain df , the number of degrees of freedom of the Student's t -distribution.

If **mn** = 1, the next element must contain the mean term b_o .

If **copts**(2) = **false**, the remaining **nreg** elements are taken as initial estimates of the linear regression coefficients b_i , for $i = 1, \dots, k$.

10: **hp – double scalar**

If **copts**(2) = **false**, **hp** is the value to be used for the pre-observed conditional variance; otherwise **hp** is not referenced.

11: **copts(2) – logical array**

The options to be used by g13fa.

copts(1) = **true**

Stationary conditions are enforced, otherwise they are not.

copts(2) = **true**

The function provides initial parameter estimates of the regression terms, otherwise these are to be provided by you.

12: **maxit – int32 scalar**

The maximum number of iterations to be used by the optimization function when estimating the GARCH(p, q) parameters. If **maxit** is set to 0 then the standard errors, score vector and variance-covariance are calculated for the input value of θ in **theta**; however the value of θ is not updated.

Constraint: **maxit** ≥ 0 .

13: **tol – double scalar**

The tolerance to be used by the optimization function when estimating the GARCH(p, q) parameters.

5.2 Optional Input Parameters

1: **num** – int32 scalar

Default: The dimension of the arrays **yt**, **et**, **ht**. (An error is raised if these dimensions are not equal.)

T , the number of terms in the sequence.

Constraint: **num** $\geq \max(\mathbf{ip}, \mathbf{iq})$.

2: **npar** – int32 scalar

Default: The dimension of the arrays **theta**, **se**, **sc**, **covar**. (An error is raised if these dimensions are not equal.)

the number of parameters to be included in the model. **npar** = 1 + **iq** + **ip** + **isym** + **mn** + **nreg** when **dist** = 'N', and **npar** = 2 + **iq** + **ip** + **isym** + **mn** + **nreg** when **dist** = 'T'.

Constraint: **npar** < 20.

5.3 Input Parameters Omitted from the MATLAB Interface

ldx, ldcovr, work, lwork

5.4 Output Parameters

1: **theta(npar)** – double array

The estimated values $\hat{\theta}$ for the vector θ .

The first element contains the coefficient α_o , the next **iq** elements contain the coefficients α_i , for $i = 1, \dots, q$.

The next **ip** elements are the coefficients β_j , for $j = 1, \dots, p$.

If **isym** = 1, the next element contains the estimate for the asymmetry parameter γ .

If **dist** = 'T', the next element contains an estimate for df , the number of degrees of freedom of the Student's t -distribution.

If **mn** = 1, the next element contains an estimate for the mean term b_o .

The final **nreg** elements are the estimated linear regression coefficients b_i , for $i = 1, \dots, k$.

2: **se(npar)** – double array

The standard errors for $\hat{\theta}$.

The first element contains the standard error for α_o . The next **iq** elements contain the standard errors for α_i , for $i = 1, \dots, q$. The next **ip** elements are the standard errors for β_j , for $j = 1, \dots, p$.

If **isym** = 1, the next element contains the standard error for γ .

If **dist** = 'T', the next element contains the standard error for df , the number of degrees of freedom of the Student's t -distribution.

If **mn** = 1, the next element contains the standard error for b_o .

The final **nreg** elements are the standard errors for b_j , for $j = 1, \dots, k$.

3: **sc(npar)** – double array

The scores for $\hat{\theta}$.

The first element contains the score for α_o . The next **iq** elements contain the score for α_i , for $i = 1, \dots, q$. The next **ip** elements are the scores for β_j , for $j = 1, \dots, p$.

If **isym** = 1, the next element contains the score for γ .

If **dist** = 'T', the next element contains the score for df , the number of degrees of freedom of the Student's t -distribution.

If **mn** = 1, the next element contains the score for b_o .

The final **nreg** elements are the scores for b_j , for $j = 1, \dots, k$.

4: **covr(ldcovr,npar) – double array**

The covariance matrix of the parameter estimates $\hat{\theta}$, that is the inverse of the Fisher Information Matrix.

5: **hp – double scalar**

If **copts(2)** = **true**, **hp** is the estimated value of the pre-observed of the conditional variance.

6: **et(num) – double array**

The estimated residuals, ϵ_t , for $t = 1, \dots, T$.

7: **ht(num) – double array**

The estimated conditional variances, h_t , for $t = 1, \dots, T$.

8: **lgf – double scalar**

The value of the log-likelihood function at $\hat{\theta}$.

9: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g13fa may return useful information for one or more of the following detected errors or warnings.

ifail = 1

On entry, **nreg** < 0,
 or **mn** > 1,
 or **mn** < 0,
 or **isym** > 1,
 or **isym** < 0,
 or **iq** < 1,
 or **ip** < 0,
 or **npar** ≥ 20,
 or **npar** has an invalid value,
 or **ldcovr** < **npar**,
 or **ldx** < **num**,
 or **dist** ≠ 'N',
 or **dist** ≠ 'T',
 or **maxit** < 0,
 or **num** < max(**ip**, **iq**).

ifail = 2

On entry, **lwork** < (**nreg** + 3) × **num** + **npar** + 403.

ifail = 3

The matrix X is not full rank.

ifail = 4

The information matrix is not positive-definite.

ifail = 5

The maximum number of iterations has been reached.

ifail = 6

The log-likelihood cannot be optimized any further.

ifail = 7

No feasible model parameters could be found.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

```
nt = int32(4);
iseed = zeros(4, 1, 'int32');
iseed(1) = int32(111);
igen = int32(0);
rwsav = zeros(9, 1);
rvec = zeros(40, 1);
num = int32(1500);
param = [0.8; 0.6; 0.2; 0.1];
mean = 3;
bx = [1.5; 2.5; 3];
df = 4.1;
dist = 'N';
gamma = -0.4;
bx = [1.5; 2.5; 3];
copts = [true; true];
ip = int32(0);
iq = int32(3);
nreg = int32(0);
mn = int32(1);
isym = int32(1);
theta = [0.4;
         0.3;
         0.1;
         0.05;
         -0.2;
         0];
hp = 0;
maxit = int32(200);
tol = 1e-05;
x = zeros(num, 3);
for i = 1:num
    fac1 = double(i)*0.01;
    x(i,1) = 0.01 + 0.7*sin(fac1);
    x(i,2) = 0.5 + fac1*0.1;
    x(i,3) = 1.0;
end

[igen, iseed] = g05kb(igen, iseed);
```

```

fcall = true;
[ht, yt, rvec, iseed, rwsav, ifail] = ...
    g05hk(dist, num, ip, iq, param, gamma, df, fcall, rvec, igen, iseed,
rwsav);

fcall = false;
[ht, yt, rvec, iseed, rwsav, ifail] = ...
    g05hk(dist, num, ip, iq, param, gamma, df, fcall, rvec, igen, iseed,
rwsav);

for i = 1:num
    xterm = 0;
    for k = 1:nreg
        xterm = xterm + x(i,k)*bx(k);
    end
    if (mn == 1)
        yt(i) = mean + xterm + yt(i);
    else
        yt(i) = xterm + yt(i);
    end
end

[thetaOut, se, sc, covar, hpOut, et, ht, lgf, ifail] = ...
    g13fa(dist, yt, x, ip, iq, nreg, mn, isym, theta, hp, copts, maxit,
tol)

thetaOut =
    0.8031
    0.6249
    0.1803
    0.0921
   -0.5119
    2.9860
se =
    0.0788
    0.0570
    0.0327
    0.0237
    0.0682
    0.0324
sc =
    1.0e-05 *
   -0.0676
   -0.0996
   -0.1536
   -0.1052
    0.0275
   -0.1131
covar =
    0.0062   -0.0010   -0.0006   -0.0005    0.0006    0
   -0.0010    0.0032   -0.0003   -0.0002    0.0012    0
   -0.0006   -0.0003    0.0011   -0.0002    0.0002    0
   -0.0005   -0.0002   -0.0002    0.0006   -0.0001    0
    0.0006    0.0012    0.0002   -0.0001    0.0047    0
    0         0         0         0         0    0.0011
hpOut =
    3.4560
et =
    array elided
ht =
    array elided
lgf =
   -1.6008e+03
ifail =
    0

```